ROLL PASS DESIGN FOR SHAPE ROLLING USING UPPER BOUND METHOD

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Abstract— ***This paper presents a computerised scheme for using an upper bound method for the study of external shape and torque in the single pass rolling of shaped sections. Generalized kinematic ally admissible velocity fields calculated from an assumed deforming geometry, in turn mathematically developed from a new parameterization of curves for the stream line flow of the material have been used. An algorithm based on the gradient projection method has been used for the optimization routine. An upper bound on rolling power established on the basis of the velocity fields has been used. Unknown variables in the velocity field need to be determined by minimizing rolling power with respect to unknown velocity field variables, yielding an upper limit to the actual power required as well as rolling torque. Velocity fields and power relations for oval to round pass have been used and computer analysis was carried out to analyze and simulate the process of shape rolling. The computerised scheme can be used for quick estimates of the rolling parameters.***

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I. INTRODUCTION

Since, many efforts have been made to develop methods of calculation of basic parameters for the rolling of slab, sheet, and strip. The need is for a simple and rapid method of calculating the various parameters in rolling such as roll force, torque, temperature distribution, strain-rate, strain and stress distribution. The slab method is capable of handling more realistic material behaviour such as strain and strain-rate hardening and temperature effects and is particularly appropriate for the rolling process. Approximate numerical solutions for rolling had been developed by several previous investigators, e.g. Orowan [2], Bland and Ford [3], Cook and McCrum [4], Ford, Ellis and Bland [5], Alexander and Ford [6], Lianis and Ford [7], and Sims [8], as discussed in the review by Ford [9].

The most comprehensive of these earlier theories is that of Orowan [2], who developed a 'homogeneous graphical method' of solution, including an attempt to allow for the inhomogeneity of deformation throughout the volume of the plastically deforming material in the arc of contact. All other theories, including the original basic theory of von Karman [10] assumed that 'plane sections remain plane' during passage of the strip through the arc of contact. Also, apart from Orowan, all other researchers had used approximations such as $\sin\varphi \approx \tan\varphi$ and $(1-\cos\varphi) = 0$ or $\varphi^2/2$ and that a mean flow stress could be used through the contact arc. This was understandable, in view of the complexity of Orowan's homogeneous graphical method.

With the advent of modern computers, Alexander [11] showed how any rolling problem could be solved accurately using the same basic approach as that developed by Orowan. Unlike Orowan's method, however, no allowance was made in the computer program for inhomogeneous deformation through the arc of contact. Since the inhomogeneity factor introduced by Orowan can at best be only an approximation to the real situation of unknown accuracy it does not appear worthwhile to try to include it in the computer solution. It has been shown conclusively by Abd-Rabbo and Venter [12] that the effect is very small, in any case. The only way in which a true prediction of the in homogeneity of deformation can be achieved is either by development of slip line field solutions with their approximations or by using finite element methods, also prone to considerable approximations unless very fine meshes requiring large computer capacity can be employed.

Lahoti et al [13] have modelled the rolling process using this upper bound method. They used Hill's [14] kinematically admissible velocity field to derive expressions for the strain-rate components $\dot{\varepsilon}_x$, $\dot{\varepsilon}_y$ and $\dot{\varepsilon}_z$ and the effective strain-rate $\dot{\varepsilon}$. Strain-rate, flow stress *(σ)* and shear stress (τ) were used to calculate the total energy dissipation rate $E_{T.}$

Many researchers, have modelled the rolling process using the finite-element method. The basis of finite-element modelling, using the variational approach, is to formulate proper functionals, depending upon specific constitutive relations. Kobayashi et ale [15]), took the initiative to propose the "Complex Element Method", combining the concept of the rigid plastic 3-D FEM and the slab method. Soon after that a more simplified method was proposed by Kimet et. al. [16], a combined slab method with rigid plastic 2-D FEM. Both of the above two methods divide the billet into several slabs, and process the FEM analysis. A great deal of CPU time is indeed saved; however, the precision is lowered for shape rolling.

The development of an analytical, theoretically based method to predict metal flow, roll torque, rolling pressure and force, and exit velocity for shape rolling is of great benefit to the rolling industry. Such a method has been presented in this study for the analysis of rolling of simple geometrical shapes of oval and round sections. The analysis assumes elastic deformations to be negligible and does not consider combined thermo-mechanical effects. A parametric formulation has been proposed to cover the entire deformation zone, based on which strain rates and velocities are determined. The oval-to-round and diamond-to-rectangle passes have been analyzed with the new method.

2. Mathematical model

The material under the action of the rolls deforms from an original shape (oval in this paper) to a final shaped section (round). This deformation happens in a bounded region, in which the initial and final sections are assumed to have arbitrary shape and considered as plane surfaces. The other surfaces of the deforming region are the roll surfaces.

For metal forming operations, no exact solutions may be available for the variables, such as load, pressure, velocities, etc. involved in plastic deformation. Two methods have been developed to establish these parameters: the upper bound method, which is certainly an over estimate and the lower bound method, which is an underestimate. Upper bound methods are valuable tools in solving metal forming problems since an upper bound ensures a conservative effect. The upper bound method involves construction of a kinematically admissible velocity field for a deformation process and a simultaneous minimization of energy provides the so-called "best solution" for the process. To accurately model the rolling process the relative velocity between the rolls and the workpiece needs to be evaluated which centers around an upper bound solution to predict the velocity field. Once again, plane strain conditions are assumed for the problem.

An upper bound method of analysis was selected for the present study because this approach can provide useful rolling mill design data for the more significant aspects of the rolling problem. The method is more accurate than the slab method than an approach based on the finite-element method of analysis. The application of the upper bound method to problems in roll pass design has been described by Abrinia and Fazlirad[1]. In this paper, a computer code has been developed for implementing this method.

The first step in the method is to define streamlines that establish an envelope for the deformation zone, an envelope which completely defines the shape of the plastically deforming body. In order to calculate the coordinates of the billet surface streamlines it is necessary to define the streamline equation, an equation which actually define the slope of the pathlines of material points as they flow through the deformation zone. The streamline equations are expressed in terms of velocity field.

Bezier curve is used to mathematically describe the streamline. The Bezier curve has a number of properties which is convenient to design a curve. In general, a Bezier curve can be fitted to any number of control points. The number of control points to be approximated and their relative position decide the degree of the Bezier polynomial. A Bezier curve can be specified with boundary conditions, with a characterizing matrix, or with a blending functions [17]. The blending function specification is the most convenient. A Bezier curve of degree n is defined in parametric form by the following equation:

$$P(t) = \sum_{i=0}^{n} P_i\, B(t)_{i,n} \qquad 0 \le t \le 1$$

Where there are n+1 control points, t is a single value parameters varying from 0 to 1, and P_i is a vector of coordinates of the control point i.e. $P_i = \{x_i, y_i\}$ and $B(t)_{i,n}$ is the blending functions between P_0 and P_n.

The shape of the Bezier curve is determined by position vectors of the control points. The curve passes through the first and last control points. Shape and hence degree of the polynomial can be changed by adding and deleting data points.

Expressing streamlines within the deformation zone as Bezier curves require appropriate definition of position vectors.

In the upper bound-method of analysis an admissible velocity field solution is considered through an assumption of the shape of the streamlines in the deformation zone.. Based on this velocity field, the total energy dissipation rate for the metal deformation rate necessarily predicts a higher value than that required in the actual metal deformation process, based on the limit theorems, the lower the energy dissipation rate, the better the prediction. Thus, the metal flow distribution can be determined by minimizing the energy dissipation rate with respect to the velocity field with the actual minimization performed with respect to certain unknown parameters which are introduced in order to represent the metal flow in the actual process. Although increasing the number of unknown parameters in the velocity field will generally improve the solution, the computations become more complex. Consequently, in the upper bound method of analysis practical compromises must be made in choosing an admissible velocity field.

The total power *W* required to deform the material in a rolling operation is expressed as the sum of the following components: the power required to deform the material plastically, W_i; the power required to overcome shear forces resulting from velocity discontinuities at entry W_e, at exit W_x; and the power required to overcome friction, W_f. In mathematical terms:

$$W = W_i + W_e + W_x + W_f$$

Where,

$$W_i = \bar{\sigma} \int \left(\frac{2\varepsilon_{ij}\varepsilon_{ij}}{3}\right)^{1/2}$$

$$W_e = \bar{\sigma} \int V_1\, dA_1$$

$$W_x = \bar{\sigma} \int V_2\, dA_2$$

$$W_f = \frac{m}{\sqrt{3}}\, \bar{\sigma} \int V_f\, dA_f$$

The relation for power includes seven unknown coefficients; a_1 to c_2 and Ve which will be determined by the optimization of the total power required. A C-computer program utilizing the Algorithm was developed to perform the optimization. W_i was calculated by numerical integration with respect to each of the parameters *u*, *q*, and *t*. For W_e and W_x, analytical integration with respect to u was followed by numerical integration with respect to q. For W_i, integration was performed numerically with respect to both parameters.

SIMULATION OF PASS SEQUENCE

While the method described earlier is applicable to different roll pass sequences, the computer procedure reported in this paper addresses only the oval to round sequence.

A schematic illustration of the shape rolling process considered in the present study is shown in Fig. below. It is assumed that a pair of rigid grooved rolls, with outer diameter and outside tangential roll velocity V_r, draw the billet into the roll groove and reduce the billet cross sectional area. It is assumed that the entry plane of the deformation zone is a plane perpendicular to the direction of rolling located at the initial touching point between the billet entry cross section and the roll surfaces. It is also assumed that the exit plane of the deformation zone is a plane perpendicular to the direction of rolling located within the plane which contains the roll centrelines. The projected length of the roll gap and the distance between the entry and exit planes z_0, is also determined from the specified billet entry cross section and the geometric parameters of the groove design.

In the shape rolling analysis an analytical representation of the three dimensional roll groove surface is required. This surface is a surface of revolution which can be defined analytically in terms of a function f(x), a function of x and y coordinates, which expresses the z coordinate values of points on the surface. This function can be expressed as follows:

$$(y_c - f(x))^2 = (y - y_c)^2 + (z_0 - z)^2$$

y and z are the coordinates of a rotated point on the roll groove surface.

For the production of round bars the oval-to-round pass is the most widely used rolling sequence. In Figure below, a quarter of the deformation zone for an oval billet to a round bar is shown. Lower roll has been omitted for clarity. The rolling geometry is assumed to be symmetrical.

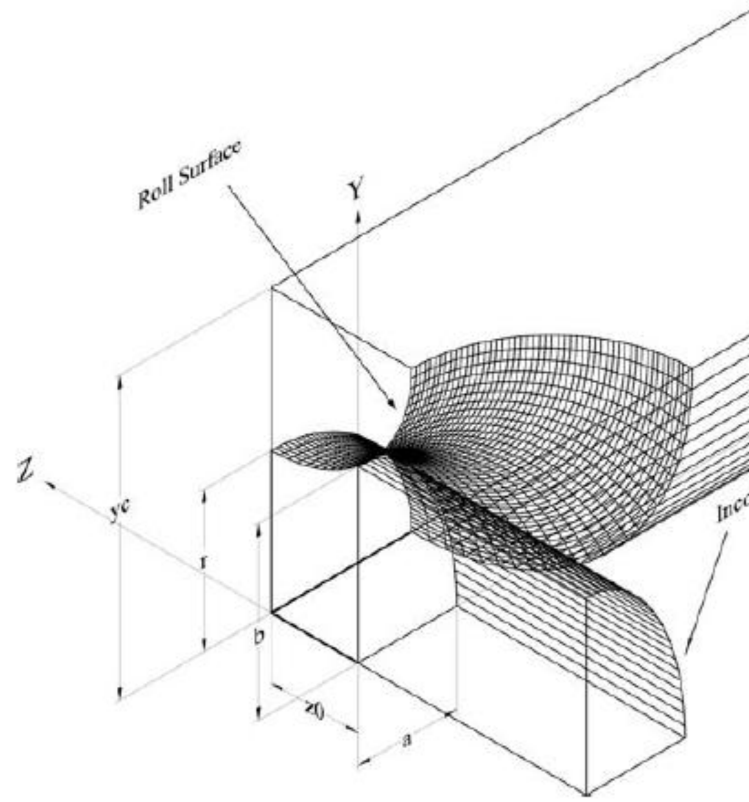

Fig.1. A quarter of the oval to round pass deformation zone [1]

In Fig.1, left hand side coordinate system has been employed to simplify relation. Length 2a and 2b is the major and minor axes of an ellipse respectively, at the entry cross section for the oval to round pass. The exit cross section is a circle with radius r.

For the purpose of this study, a single Bezier curve was found to be sufficient to appropriately describe the entire deformation zone.

Based on, the parametric Bezier formulation for any streamline, within the deformation zone in the oval-to-round pass has been expressed [1] as follows:

$$\vec{V_0} = ua\,\sin\varphi\,\bar{\imath} + ub\,\cos\varphi\,\bar{\jmath}$$
$$\vec{V_1} = a_1 ua\,\sin\varphi\,\bar{\imath} + b_1 ub\,\cos\varphi\,\bar{\jmath} + c_1 z_0 \bar{k}$$
$$\vec{V_2} = a_2 ur\,\sin\varphi\,\bar{\imath} + b_2 ur\,\cos\varphi\,\bar{\jmath} + c_2 z_0 \bar{k}$$

$\vec{V_3} = ur\ \sin\varphi\ \bar{\imath} + ur\ \cos\varphi\ \bar{\jmath} + z_0\bar{k}$

where $\varphi = \pi q/2$ describes the angular position of the control point $\vec{V_3}$.

The Bezier curve may be expressed in parametric terms as

$\vec{r}(u, q, t) = (1-t)^3\vec{V_0} + 3t(1-t)^2\vec{V_1} + 3t^2(1-t)\vec{V_2} + t^3\vec{V_3}$

or in Cartesian form:

$\bar{r}(u,q,t) = f\ (u,q,t)\bar{\imath} + g(u,q,t)\bar{\jmath} + h(u,q,t)\bar{t}$

The Cartesian form has been used in the computer code. Vector $\bar{r}$ provides a parametric definition of all the streamlines in the deformation zone. Velocities in three dimensions were found.

ADMISSIBLE VELOCITY FIELDS

The most difficult step in applying the upper-bound method of analysis is to develop a class of admissible velocity fields. The admissible velocity is a set of functions which satisfies the incompressibility condition, the essential velocity boundary conditions in the roll work piece interface, and continuity across elastic plastic interfaces within the plastically deforming body.

The incompressibility condition may be expressed as follows,

$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0$

V_x , V_y V_z are the components of the three dimensional velocity vector along coordinate axes X, Y and Z, respectively.

MINIMIZATION VARIABLES

In the shape rolling analysis there are seven unknown parameters in the velocity field equations: a_1, a_2, b_1, b_2, c_1, c_2 and ve. Hence, the total energy dissipation rate, can be expressed as a function of these unknown parameters. Hence, the total energy dissipation rate can be expressed as a function of these unknown parameters, as follows,

$$W_{tot} = W_{tot}\ (a_1, a_2, b_1, b_2, c_1, c_2, ve)$$

The six unknown parameters a_1, a_2, b_1, b_2, c_1, c_2 determine the points on the streamline and Ve determine the billet velocity upon entry to the deformation zone.

The total energy dissipation rate W_{tot}, is formulated from the velocity field and the strain rate equations are expressed as follows,

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$$

$$\dot{\varepsilon}_{xx} = \left(\frac{\partial v_i}{\partial x_i}\right)$$

$$\dot{\varepsilon}_{yy} = \left(\frac{\partial v_j}{\partial x_j}\right)$$

$$\dot{\varepsilon}_{zz} = \left(\frac{\partial v_k}{\partial x_k}\right)$$

$\dot{\varepsilon}_{xx}, \dot{\varepsilon}_{yy}\ and\ \dot{\varepsilon}_{zz}$ are substituted in the incompressibility condition i.e. $\dot{\varepsilon}_{ii} = 0$.

Using the parametric notation employed in this study, the plastic deformation component of the total rolling power has been expressed as follows:

$$W_i = \frac{2}{\sqrt{3}}\bar{\sigma}\int\left(\frac{\dot{\varepsilon}_{xx}^2 + \dot{\varepsilon}_{yy}^2 + \dot{\varepsilon}_{zz}^2}{2} + \dot{\varepsilon}_{xy}^2 + \dot{\varepsilon}_{yz}^2 + \dot{\varepsilon}_{zx}^2\right)^{1/2}|J|\,dudqdt$$

$\dot{\varepsilon}_{xx}$ ….. are the strain rates in various directions, $\bar{\sigma}$ is the flow stress and $|J|$ is the Jacobian for transformation of coordinates from x, y and z to u, q and t.

Plastic deformation component W_i is integrated between limits 0 to 1. It is determined by the entry plane of the deformation zone, exit plane of the deformation zone, roll workpiece and billet free side surfaces. Using numerical integration with respect to each of the parameters u, q, and t plastic deformation component W_i is calculated.

The velocity discontinuity components of total rolling power can be expressed as follows. At entry:

$$W_e = \frac{\bar{\sigma}}{\sqrt{3}}\int_0^1\int_0^1 (V_x^2 + V_y^2)^{1/2}\frac{\partial(x,y)}{\partial(u,q)_{t=0}}\,dudq$$

In the entry plane there is no deformation and W_e at t = 0 is calculated by analytical integration with respect to u was followed by numerical integration with respect to q.

At exit:

$$W_x = \frac{\bar{\sigma}}{\sqrt{3}}\int_0^1\int_0^1 (V_x^2 + V_y^2)^{1/2}\frac{\partial(x,y)}{\partial(u,q)_{t=1}}\,dudq$$

In the exit plane W_x at t = 1 is calculated by analytical integration with respect to u was followed by numerical integration with respect to q. The friction component:

$$W_f = \frac{m\bar{\sigma}}{\sqrt{3}} \sqrt{(V_t - V_r) + V_y^2}\, \sec\alpha \frac{\partial(x,y)}{\partial(q,t)_{u=1}}\, dq\, dt$$

For the energy dissipation rate due to friction W_f, which is integrated over the roll work piece interface, the friction shear factor m, is given an input value

Where $\sec\alpha = \frac{\sqrt{N_1^2 + N_2^2 + N_3^2}}{N_2}$ and $\vec{N} = \frac{\partial \bar{r}}{\partial q_{u=1}} X \frac{\partial \bar{r}}{\partial t_{u=1}}, V_t = \sqrt{V_x^2 + V_y^2}$ and V_r is the tangential roll velocity.

Friction component is integrated analytically with respect to the y coordinate, and numerically with respect to the x coordinate.

In the shape rolling analysis, because of the increased complexity of the shape rolling velocity field W is integrated numerically the corresponding expressions for Wi, We, Wx and Wf.

CALCULATION PROCEDURE

The calculations for the deformation process involves the following steps :

1. Input of Fixed data i.e. half length of major and minor axes of billet cross section in oval to round rolling pass, projected length of deformation zone, diameter of exit cross section in oval to round rolling pass
2. Assume some trial value of the parameters a_1, a_2, b_1, b_2, c_1, c_2 and v_e (defined earlier)
3. Calculate the strain rate $\dot{\varepsilon}_{xx}, \dot{\varepsilon}_{yy}, \dot{\varepsilon}_{zz}, \dot{\varepsilon}_{xy}, \dot{\varepsilon}_{yz}, \dot{\varepsilon}_{zx}$ and the Jacobian matrix using

$$|J| = \frac{\partial(x,y,z)}{\partial(u,q,t)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial q} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial q} & \frac{\partial y}{\partial t} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial q} & \frac{\partial z}{\partial t} \end{vmatrix}$$

   $|J|$ is the Jacobian for transformation of coordinates from x, y, z to u, q, t.

4. Calculate of V_x, V_y and V_z at $t=0$ and $t=1$.
5. Calculate $\frac{\partial(x,y)}{\partial(u,q)}$ at $t=0$ and $t=1$. (using a numerical procedure)
6. Calculate V_t , $V_r \frac{\partial(x,z)}{\partial(q,t)}$ at $u=1$.
7. Calculate W_i , W_e , W_x , W_f (using a numerical procedure for integration).
8. Calculate $W = W_i + W_e + W_x + W_f$
9. Use an optimization procedure to calculate the values of a_1, a_2, b_1, b_2, c_1, c_2 and v_e that will minimize W (the optimization procedure is described below)
10. The value of W corresponding to the optimum point gives the rolling power. This value is used to compute the rolling torque and roll separating force
11. The values of a_1, a_2, b_1, b_2, c_1, c_2 corresponding to the optimum point define the actual streamlines
12. With these values the shape of the deformed workpiece at any intermediate point between the entry and exit planes can be calculated

OPTIMIZATION ALGORITHM

Rosen's Gradient Projection Method of optimization is used to minimize the total energy rate equation which is used to determine the constants a_1, a_2, b_1, b_2, c_1, c_2 and ve. The gradient projection method of Rosen [18] does not require the solution of an auxiliary linear optimization problem to find the usable feasible direction. It uses the projection of the negative of the objective function gradient onto the constraints that are currently active. Although the method has been described by Rosen for a general nonlinear programming problem, its effectiveness is confined primarily to problems in which the constraints are all linear, which is the case in this problem added in this paper. The optimization method proceeds by a sequence of steps using an initially specified value. If the step does not contain the nonnegative components then that value is the optimum value. If the step is not a success the negative component is optimised i.e. the procedure is repeated again.

ALGORITHM BASED ON ROSEN'S GRADIENT PROJECTION METHOD

The procedure involved in the application of the gradient projection method can be described by the following steps

1. Start with an initial point X_1. The point X_1 has to be feasible, that is,
2. Set the iteration number as *i=1.*
3. If X_i is an interior feasible point [i.e., if $g_j(X_i) < 0$ for j=1, 2, ... m]set the direction of search as $S_i = \nabla f(X_i)$, normalize the search direction as

   $$Si = \frac{-\nabla f\ (Xi)}{\|\nabla f\ (Xi)\|}$$

   And go to step (5). However, if $g_j(X_i) = 0$ for $j_1, j_2, \ldots j_p$, go to step 4
4. Calculate the projection matrix P_i as

   $$Pi = I - N_p(N_p^T N_p)^{-1}N_p^T$$

   Where

   $$N_p = [\nabla g_{j1}(X_i) \quad \nabla g_{j2}(X_i) \quad \nabla g_{j2}(X_i) \quad . \quad . \quad . \quad \nabla g_{jp}(X_i)]$$

   And find the normalized search direction S_i as

   $$Si = \frac{-Pi\ \nabla f\ (Xi)}{\|\mathrm{Pi}\ \nabla f\ (Xi)\|}$$

5. Test whether or not $S_i = 0$. If $S_i \neq 0$, go to step 6. If $S_i = 0$ compute the vector λ at X_i as

   $$\lambda = (N_p^T N_p)^{-1}N_p^T\nabla f\ (Xi)$$

   If all the components of the vector λ are nonnegative, take $x_{opt} = x_i$ and stop the iterative procedure. If some of the components of λ are negative, find the component λ_q that has the most negative value and from the new matrix N_p as

   $$N_p = [\nabla g_{j1} \quad \nabla g_{j2} \ . \ . \ . \ \nabla g_{jq-1} \ \ \nabla g_{jq+1} \ . \ . \ . \ \ \nabla g_{jp}]$$

   and go to step 3.
6. If $S_i \neq 0$, find the maximum step length λ_m that is permissible without violating any of the constraints as $\lambda_m = \min(\lambda_k), \lambda_k > 0$ and k is any integer among 1 to m other than $j_1, j_2, \ldots j_p$. Also find the value of $\frac{\partial f}{\partial \lambda(\lambda_m)} = S_i^T \nabla f\ (Xi + \lambda_m S_i)$. If $\frac{\partial f}{\partial \lambda(\lambda_m)}$ is zero or negative, take the step length as $\lambda_i = \lambda_m$. On the other hand, if $\frac{\partial f}{\partial \lambda(\lambda_m)}$ is positive, find the minimizing step length λ_i*. Either by interpolation or by any of the methods discussed in Chapter 5 and take $\lambda_i = \lambda_i$*.
7. Find the new approximation to the minimum as

   $$X_{i+1} = X_i + \lambda_i S_i$$

8. If $\lambda_i = \lambda_M$ or if $\lambda_M < \lambda_i$ *,some new constraints (one or more) become active at X_{i+1} and hence generate the new matrix N_p to include the gradients of all active constraints evaluated at X_{i+1}. Set the new itration number as *i* = *i* + 1. And go to step 4. If $\lambda_i = \lambda_i *$ and , $\lambda_i * < \lambda_M$ no new constraint will be active at X_{i+1} and hence

the matrix N_p remains unaltered. Set the new value of *i* as $i = i + 1$, and go to step 3.

With the aid of a digital computer, using the upper bound method requires the total energy to be minimized.

OPTIMIZATION ALGORITHM

Rosen's Gradient Projection Method of optimization is used to minimize the total energy rate equation which is used to determine the constants a_1, a_2, b_1, b_2, c_1, c_2 and *ve*. The gradient projection method of Rosen [18] does not require the solution of an auxiliary linear optimization problem to find the usable feasible direction. It uses the projection of the negative of the objective function gradient onto the constraints that are currently active. Although the method has been described by Rosen for a general nonlinear programming problem, its effectiveness is confined primarily to problems in which the constraints are all linear, which is the case in this problem added in this thesis. The optimization method proceeds by a sequence of steps using an initially specified value. If the step does not contain the nonnegative components then that value is the optimum value. If the step is not a success the negative component is optimised i.e. the procedure is repeated again.

OPTIMIZATION SOLUTION

Table 1. Analytical conditions for a single oval to round sequence

Oval to round pass sequence	Analytical conditions
Entry cross section dimensions	2a=23.8 mm and 2b= 32.9 mm
Exit cross section dimensions	R=12.7 mm
Area reduction	17%
Roll gap	1.58
Roll tangential velocity	254 mm/s
Rotational speed	105rpm

Since, entry and exit of the deformation zone is known. On applying boundary conditions, we get the constraints for optimization. Quarter of the deformation zone is taken, on applying boundary conditions, we get 15 constraints and 7 unknown variables a_1, a_2, b_1 b_2, c_1, c_2 and v_e.

RESULTS

The upper bound analysis described by Abrinia and Fazlirad [1] for the oval to round passes has been used to develop a computer based calculation procedure for calculations related to the deformation process and the rolling power.

ESTIMATION OF MATERIAL FLOW STRESS

The flow stress, of a given metal alloy is influenced by factors related to the deformation process, such as, temperature of deformation, the strain and strain rate.

In hot rolling, defined as rolling temperature above the recrystallization temperature, the flow stress can be considered to be primarily a function of temperature and strain rate. In a single pass rolling process, the temperature and strain rate do not vary significantly within the plastically deforming body. Thus, in hot rolling the flow stress can be approximated as being a constant value within the plastically deforming body, and, therefore, it is reasonable to use the upper bound method of analysis to analyze the metal deformation. Actually, in practical situations, it is only necessary to estimate the material flow stress after the metal deformation is determined. By then the flow stress can be estimated from average values of the temperature of deformation, strain, and strain rate, quantities which are, themselves, already known or determined from the metal deformation analysis in the upper bound approach.

On the other hand, in cold rolling, at or room temperature, the flow stress of most metal alloy is primarily a function of strain. Therefore, the application of the upper bound method of analysis in cold rolling must be approximated because the material flow stress could vary significantly within the plastically deforming body, and under these conditions, the assumption of constant flow stress could introduce errors in the metal deformation analysis.

A widely accepted analytical representation of flow stress data in metal forming is in the form of empirical equation. The material is low carbon steel AISI 1018 and rolling was performed at 1090' in which the flow stress may be described by Kennedy[20].

$$\dot{\bar{\varepsilon}} = 77.5\dot{\bar{\varepsilon}}^{0.192}\text{MPa}$$

Where $\dot{\bar{\varepsilon}}$ is the effective strain rate as

$$\dot{\bar{\varepsilon}} = \left(\frac{2}{3}\,\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}\right)^{\frac{1}{2}}$$

Angular velocity, w

$$w = 2\pi N/60$$

Roll tangential velocity, V

$$V = Rw$$

$$Z_0^2 = R^2 - (R - (2b - 2r)/2)^2$$

With the minimum value of the total energy dissipation rate, the roll torque can then be computed by equating the rate of work input required to drive the rolls with the total energy dissipation, W. Thus, the roll torque was calculated in various roll pass dimensions to incorporate several area reduction values,

Area reduction is given by:

$$Q = \left(1 - \frac{A_{ex}}{A_e}\right)$$

Roll torque is given by:

$$Torque = \frac{W*60}{2\pi N}$$

Table 2. Roll torque in various area reduction

Reduction in area	Roll torque			
	z/z0=0.25	z/z0=0.5	z/z0=0.75	z/z0=1
0.015	4.23	4.49	4.6	5.08
0.04	5.79	6.89	7.6	9.036
0.1.6	5.67	7.63	10.67	12.98
0.176	6.57	9.036	12.96	15.95

CONCLUSION

In this work, a method has been proposed to analyze external shape, material flow stress and roll torque for shape rolling. This method employs an upper bound solution coupled with a numerical optimization of the total power required to deform the material and in the process determines unknown variables. This method is based on the parametric curve formulation to define streamlines within the deformation zone. Kinematically admissible velocity field were derived from the parametric curve formulation, and upper bound was established on the rolling power required. In this, step by step computer code for power has been developed.

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