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# Stability Evaluation of Interval-Valued Infectious Disease Models Employing Fourier and Laplace Transforms

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**Abstract**
The comprehension and regulation of the transmission of diseases within populations are contingent upon the stability analysis of interval-valued infectious disease models. Interval-valued models are particularly beneficial in capturing the inherent uncertainties and variations in disease parameters, thereby providing a more robust framework for analyzing infectious disease dynamics. Fourier and Laplace transforms are employed to determine the conditions for the stability of equilibrium points in interval-valued infectious disease models. The Fourier transform is employed to analyze the frequency domain characteristics of the model, thereby facilitating the identification of potential oscillatory behavior and stability issues. In the interim, the Laplace transform is implemented to evaluate the stability of the time-domain by transforming differential equations into algebraic equations, thereby streamlining the stability analysis process. The findings suggest that the stability of interval-valued infectious disease models is substantially impacted by the interaction between parameter intervals and various disease compartments. Explicit stability criteria are developed and their implications for disease control strategies are assessed. Both transforms are effective instruments for assessing stability. The Fourier transform provides a comprehensive perspective on dynamic stability, whereas the Laplace transform offers insights into periodic behavior. The field of infectious disease modeling by extending traditional methodologies to incorporate parameter uncertainties and providing practical stability criteria for interval-valued models. These advancements have the capacity to improve the design of control measures and increase the reliability of predictions in public health interventions.
**Keywords:** Interval-Valued Models, Infectious Disease Dynamics, Stability Analysis, Fourier Transform, Laplace Transform, Parameter Uncertainty

## 1. Introduction

The stability evaluation of infectious disease models, particularly those with uncertain parameters like interval values, can be approached through various mathematical techniques. Studies have delved into stability analyses of infectious disease models using methods such as Fourier and Laplace transforms [1] [2]. These analyses often involve investigating the stability of disease-free equilibrium points and endemic equilibrium points, considering factors like treatment rates, infection rates, and the impact of interventions like lockdowns and vaccinations [3]. By employing numerical simulations and theoretical frameworks, researchers have contributed to understanding the dynamics of infectious diseases, identifying conditions for stability, and exploring the effects of random fluctuations, waning immunity, and subclinical infections on the spread and control of diseases like COVID-19 [4] [5].

Infectious diseases continue to be a significant concern for global health systems, and it is imperative to employ effective modeling to comprehend and regulate their dissemination. The inherent uncertainties and variability in disease dynamics can be oversimplified by traditional models, which frequently depend on fixed parameters. Interval-valued infectious disease models provide a more adaptable approach by representing parameters as ranges rather than precise values, thereby preserving the uncertainty and variability that are inherent in real-world data. The robustness of predictions is enhanced by the ability to estimate a wider range of parameters in these interval-valued models. It is imperative to assess the stability of these models to guarantee that they can consistently manage uncertainties and continue to generate precise forecasts. Stability analysis is a valuable tool for assessing the impact of minor parameter modifications on the model's behavior and predictions. Advanced mathematical techniques, specifically Fourier and Laplace transforms, are implemented in this investigation to evaluate the stability of interval-valued infectious disease models. Laplace transforms are effective for managing complex differential equations and system dynamics, while Fourier transforms are beneficial for examining periodic and oscillatory behaviors within the model.

### 1.1 Overview of infectious disease models.

Infectious disease models play a crucial role in understanding and managing the spread of diseases in both human and wildlife populations. Mathematical models are extensively used to explain, predict, and make decisions related to disease control and resource management [6] [7]. These models help conceptualize host-pathogen interactions, identify key components, and evaluate the consequences of different management strategies, including the development of vaccines and therapeutics [8]. Various types of models, such as single-host and multihost models, are employed to assess disease dynamics and calculate essential parameters like the basic reproductive number ($R_0$), which is pivotal in epidemiology for determining disease emergence and control strategies. Despite the challenges of reproducibility between animal and human studies, animal models remain valuable tools for investigating infectious diseases and advancing our understanding of host immune responses, pathogenesis, and microbiome-pathogen interactions.

The stability evaluation of interval-valued infectious disease models using Fourier and Laplace transforms involves analyzing how the dynamics of disease spread are influenced by varying parameters within given intervals. These models are particularly useful in capturing uncertainties and variability in disease parameters such as transmission rates and recovery times. Fourier and Laplace transforms are mathematical tools used to convert differential equations governing the spread of infection into algebraic forms, making it easier to analyze stability. By applying these transforms, researchers can determine whether small perturbations in the system lead to stable or unstable behavior, providing insights into how resilient the disease model is to changes in parameters. This technique aids in creating efficient management plans and comprehending the long-term behavior of infectious illnesses.

### 1.2 Importance of stability analysis in epidemiological models.

Stability analysis plays a crucial role in epidemiological models as it helps in understanding the behavior and outcomes of infectious diseases. By analyzing stability, researchers can determine the conditions under which diseases will either persist or diminish, predict the dominance of specific variants in competitive epidemic processes [9], identify equilibrium points such as disease-free or endemic equilibria, and assess the effectiveness of control measures like vaccination and physical distancing [10]. Efficient stability calculations, especially in larger systems, aid in simplifying complex models and making accurate predictions [11]. Additionally, stability analysis allows for the identification of stable and unstable equilibrium states, reflecting the realistic nature of epidemiological models [12]. Overall, stability analysis is essential for providing insights into disease dynamics, guiding public health interventions, and improving epidemic control strategies.

Stability analysis in epidemiological models is crucial as it assesses the robustness of a model's predictions about the spread and control of infectious diseases. By evaluating the stability of these models, researchers can determine how small changes in parameters or initial conditions might affect the long-term behavior of disease dynamics. Stability analysis becomes even more critical in the context of interval-valued infectious disease models, which take into account uncertainties and variations in parameter estimates. Employing Fourier and Laplace transforms in this analysis allows for a more comprehensive understanding of how these interval-valued models respond to perturbations over time. Despite uncertainties in model parameters, this method assists in identifying the conditions under which the disease can be effectively managed or controlled, thereby ensuring that interventions are reliable and robust.

### 1.3 Introduction to interval-valued models and their advantages in handling uncertainty.

Interval-valued models offer a powerful approach to handling uncertainty by capturing individual measurement uncertainties, providing increased informational capacity, and enabling additional insights [13]. In situations where conventional methods based on probability density functions necessitate a large number of samples, such as the investigation of rubber nonlinear property degradation, these models are particularly advantageous [14]. By directly incorporating uncertainties in measurement data, interval-valued models enhance fault detection in industrial processes, improving monitoring performance and yielding effective results [15]. Furthermore, the integration of interval analysis with neural networks has shown promise in handling complex information, especially in non-Euclidean data spaces like graphs, where traditional models face limitations [16]. Overall, interval-valued models present a comprehensive and efficient way to address uncertainties at the individual measurement level, offering a valuable tool for various scientific and engineering applications.

Interval-valued models are a sophisticated approach used to manage and quantify uncertainty in the stability evaluation of infectious disease models. Unlike traditional models that use precise point estimates, interval-valued models operate with ranges of values, acknowledging that data and parameters may have inherent uncertainty. In the context of infectious disease models, such as those evaluating stability, this means that instead of a single fixed value, parameters like transmission rates or recovery rates are represented as intervals. Understanding the dynamics of disease transmission can be done in a more realistic and flexible manner thanks to this framework, which takes into account variability and imprecision in the data. Employing techniques like Fourier and Laplace transforms in these models allows for the analysis of system behavior in the frequency and time domains, respectively. Fourier transforms help in understanding periodic patterns and frequency components of the disease dynamics, while Laplace transforms are useful for solving differential equations and evaluating stability. By integrating interval-valued methods with these transforms, researchers can better handle uncertainties and gain more robust insights into the stability and behavior of infectious disease systems.

## 2. Literature Review

The use of interval-valued parameters has greatly benefited the study of infectious disease models by offering a more reliable framework for capturing the uncertainty present in epidemiological data. The usefulness of Fourier and Laplace transforms in evaluating the stability of these models and their capacity to address the difficulties posed by interval-valued variables.

### 2.1. Infectious Disease Models

**Bergeret.al (2020)** investigated the effects of case-dependent quarantine and testing in the SEIR infectious disease model. Asymptomatic infections were identified earlier through our model's integration of testing, which enabled the opportune quarantine of infected individuals. The initial step was to compare a variety of testing and quarantine policies, beginning with a baseline approach that focused solely on quarantine, like the practices in the United States in early 2020. The implementation of targeted quarantine and increased random testing could potentially mitigate economic impacts, reduce the number of fatalities, and reduce the peak number of symptomatic infections. To assess public health and economic policies, the model can be incorporated into more intricate SEIR models.

**Alahmadi et.al (2020)** discussed the Modern data and computational resources, coupled with algorithmic and theoretical advances to exploit these, allowed disease dynamic models to be parameterized with increasing detail and accuracy. While this enhanced the models' usefulness in prediction and policy, major challenges remained. In particular, the lack of identifiability of a model's parameters might have limited the model's usefulness. While the lack of parameter identifiability could have been resolved through the incorporation of prior knowledge into an inference procedure, formulating such knowledge was often difficult. Furthermore, there were practical challenges associated with acquiring data of sufficient quantity and quality.

**Bloom et.al (2019)** discussed the global health system was intended to protect against infectious disease hazards by instituting a variety of formal and informal networks that operated across various regions and sectors. Despite its progress, challenges continued to arise due to ongoing and emergent diseases, such as Zika and Ebola, as well as issues such as antimicrobial resistance. Disease management was further complicated by factors such as urbanization, accelerated population growth, and climate change. A multidisciplinary Global Technical Council on Infectious Disease Threats was requested to confront these obstacles. By enhancing organizational collaboration, addressing knowledge deficits, and providing evidence-based recommendations for managing infectious disease risks, this Council would have contributed to global health.

**Eikenberry et.al (2020)** proposed the use of face masks by the general public to limit COVID-19 spread was debated but increasingly recommended. A compartmental model showed that even low-efficiency masks could significantly reduce community transmission, peak hospitalizations, and deaths. In simulations for New York and Washington, broad adoption of moderately effective masks (50% efficiency) could have reduced deaths by 17–45% and peak deaths by 34–58% in New York, and in Washington, even less effective masks (20% efficiency) could have cut mortality by 24–65%. Masks were most effective when combined with other measures and used widely.

**Table 1: Comparison Table**

| Author & Year | Focus | Findings | Key Insights |
|---|---|---|---|
| Berger et al. (2020) | Effects of case-dependent quarantine and | Integration of testing identified asymptomatic infections earlier; targeted quarantine and | Targeted quarantine and testing are more effective than baseline quarantine |

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|---|---|---|---|
| | testing in SEIR model | increased random testing could mitigate economic impacts, reduce fatalities, and lower symptomatic infection peaks. | approaches; complex SEIR models can improve policy assessment. |
| Alahmadi et al. (2020) | Parameterization of disease dynamic models | Modern data and computational resources improve model accuracy; challenges include parameter identifiability and data quality. | Incorporating prior knowledge and improving data quality can enhance model usefulness; addressing parameter identifiability is crucial. |
| Bloom et al. (2019) | Global health system and infectious disease management | Ongoing challenges include emerging diseases, antimicrobial resistance, urbanization, population growth, and climate change; a Global Technical Council could address these issues. | Multidisciplinary collaboration and evidence-based recommendations are needed to effectively manage global health challenges. |
| Eikenberry et al. (2020) | Impact of face masks on COVID-19 spread | Even low-efficiency masks can significantly reduce transmission, hospitalizations, and deaths; masks are most effective with widespread adoption and combined measures. | Mask usage, even at moderate efficiency, is effective in reducing COVID-19 spread; widespread adoption and combination with other measures enhance effectiveness. |

### 2.2. Interval-Valued Models

**Strauss et.al (2022)** proposed the System modeling was used in diverse fields like chemistry, mechanics, medicine, economics, robotics, and more. A system was a real process that deterministically linked input values to output values. A model was a mathematical representation for analyzing real phenomena and predicting results. One challenge in modeling was choosing the model and measuring its accuracy. The linear model, which used a weighted sum of inputs to produce outputs, was simple and parameter-efficient but lacked precise accuracy. Non-linear models offered specificity but were complex and harder to measure for accuracy. The simplicity of linear models with an imprecise output that reflected the inadequacy of linear models for certain systems.

**Nagarajan et.al (2022)** discussed the Neutrosophic sets were implemented to address indeterminacy in practical scenarios, thereby validating their superiority in the medical world. The neutrosophic hidden Markov model effectively mitigated ambiguity, in contrast to the current hidden Markov models. Using the Viterbi algorithm to decode the optimal path in the presence of ambiguity, this study integrated single-value and interval-valued neutrosophic sets into the hidden Markov model. The neutrosophic score was employed to determine the crisp probability value, thereby reducing computation time by eliminating the necessity for a lower membership function for falsility. The potential dangers associated with childhood obesity during lockdown scenarios.

**Manna et.al (2022)** focused on the mathematical formulation of an imprecise inventory model with a partial prepayment policy in an interval environment. It considered two cases: without shortages and with partially backlogged shortages. Demand and deterioration rates, along with inventory costs (ordering, purchase, holding, shortage, and lost sale), were interval valued. The model's continuous inventory changes were defined by interval differential equations. Interval-valued average profits were calculated using a parametric approach, with selling price and business period as decision variables.

**Chacón et.al (2021)** introduced the new approaches for fitting regression models for symbolic interval-valued variables, improving and extending methods by Billard and Diday, and Lima-Neto and De Carvalho. The models used midpoints and half-lengths of intervals as additional variables and employed tree-based models, K-nearest neighbors, support vector machines, and neural networks. These methods were tested on real and synthetic datasets, using root-mean-squared error and correlation coefficient for evaluation. The methods were made available in the RSDA package in R, installable from CRAN.

**Zhang et.al (2020)** studied the demand for adaptable methods to analyze interval-valued data in large datasets was on the rise. The process by which interval-valued data were constructed, typically through the aggregation of real-valued data, was not taken into account by current descriptive frameworks. The data-generating procedure was directly integrated into the likelihood-based statistical inference for intervals that was devised. The fitting of models for the underlying real-valued data using only interval-valued summaries, thereby resolving issues such as the assumption of within-interval uniformity.

**Table 2: Comparison Table**

| **Author & Year** | **Focus** | **Findings** | **Key Insights** |
|---|---|---|---|
| Strauss et al. (2022) | System modeling across various fields including chemistry, mechanics, medicine, economics, and robotics | Proposes an imprecise linear model combining simplicity with imprecision to reflect limitations of linear models. | The imprecise linear model provides a balance between simplicity and accuracy, incorporating a convex set of outputs. |
| Nagarajan et al. (2022) | Neutrosophic sets in addressing indeterminacy, with applications in medical scenarios | Introduces a neutrosophic hidden Markov model with improved ambiguity handling using Viterbi algorithm. | Neutrosophic models enhance ambiguity resolution and efficiency in computational tasks, relevant for childhood obesity studies. |
| Manna et al. (2022) | Mathematical formulation of an imprecise inventory model with partial prepayment policy | Develops a model with interval-valued demand, deterioration rates, and costs, using interval differential equations. | Provides a method for handling interval-valued data in inventory models, validated through meta-heuristic algorithms. |
| Chacón et al. (2021) | Regression models for symbolic interval-valued variables | Introduces new approaches using midpoints and half-lengths, and various | Extends regression methods to interval-valued data, with practical implementation |

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|---|---|---|---|
| | | machine learning models for regression. | available in the RSDA package. |
| Zhang et al. (2020) | Analysis of interval-valued data in large datasets | Integrates data-generating procedures into likelihood-based inference for interval data, addressing within-interval uniformity. | Provides a framework for modeling real-valued data using interval-valued summaries, improving statistical inference. |

### 2.3. Stability Analysis

**Annas et.al (2020)** discussed the develop and assess an SEIR model for COVID-19 that would incorporate vaccination and isolation factors. The model's global stability and fundamental reproduction numbers were determined by analyzing its stability using the generation matrix method. Numerical simulations were conducted using secondary data from Indonesia. Indonesia would have continued to be endemic of COVID-19 in the absence of vaccination. It was demonstrated that vaccines could expedite recovery, while maximum isolation measures could delay the spread, and the simulation predicted future case numbers.

**Munuswamy et.al (2020)** evaluated two power factor correction (PFC) controls for single-ended primary inductance converters (SEPIC): enhanced non-linear carrier (ENLC) control and average current mode (ACM) control. The ACM control employed a linear current reference in contrast to the inductor current, whereas the ENLC control employed a non-linear carrier reference with fewer components and sensors. The investigation looked at electromagnetic interference (EMI) and nonlinear processes using power spectral density estimates. Stability and EMI analyses were conducted using MATLAB/Simulink simulations, and mathematical bifurcation analysis was employed to gain insight into the boundaries of stability.

**Huang et.al (2019)** integrated into weak power grids, they experienced grid-synchronization instability (GSI), characterized by phase-locked loop (PLL) frequency divergence and converter power output oscillations. Examined how reactive power control (RPC) methods influenced GSI. Developed a single-input-single-output model to analyze PLL interactions within the converter system and derived its open-loop and sensitivity functions. By comparing stability margins across various RPC methods, we highlighted how RPC, PLL, and voltage feedforward (VFF) design choices affected stability.

**Golbabai et.al (2019)** explored the fractional Black–Scholes model with an α-order time fractional derivative, which was used to price American and European call and put options for a non-dividend paying stock. The fractional differential equations inherent to this model, efficient numerical schemes were essential due to the non-local nature of fractional derivatives. The numerical solution of the time fractional Black–Scholes model (TFBSM) using radial basis functions (RBFs), a mesh-free method, for European option pricing. The TFBSM was discretized temporally with a finite difference scheme of order $O(\delta t^{2-\alpha})$ for $0<\alpha<1$, and spatial derivatives were approximated with RBFs. The method’s stability and convergence were theoretically demonstrated, and numerical results validated its accuracy and efficiency.

**Table 3: Comparison Table**

| Author & Year | Focus | Findings | Key Insights |
|---|---|---|---|
| Annas et al. (2020) | SEIR model for COVID-19 with vaccination and isolation factors | The SEIR model's global stability and fundamental reproduction numbers were analyzed. Vaccination expedited recovery, while maximum isolation delayed spread. Simulations predicted future case numbers. | Without vaccination, Indonesia would have continued to face COVID-19 endemically. Vaccines significantly aid in recovery, and isolation measures can delay the spread. These insights are crucial for developing early prevention strategies in Indonesia. |
| Munuswamy et al. (2020) | Power factor correction (PFC) controls for SEPIC converters: ENLC vs. ACM | ENLC control uses a non-linear carrier reference with fewer components, while ACM uses a linear reference. Stability and EMI analyses were conducted using MATLAB/Simulink. The ENLC control was experimentally verified. | ENLC control for SEPIC converters offers a simpler and potentially more effective solution compared to ACM control. The study highlights the importance of analyzing stability and electromagnetic interference in PFC controls. |
| Huang et al. (2019) | Reactive power control (RPC) methods affecting grid-synchronization instability (GSI) in weak power grids | RPC methods influenced GSI, with PLL frequency divergence and converter power output oscillations. A single-input-single-output model was developed for analysis, showing the impact of RPC, PLL, and VFF on stability. | Different RPC methods and design choices affect stability margins and the likelihood of grid-synchronization instability. Understanding these interactions is crucial for improving stability in weak power grids. |
| Golbabai et al. (2019) | Fractional Black–Scholes model for option pricing using radial basis functions (RBFs) | The fractional Black–Scholes model (TFBSM) was solved using RBFs and finite difference schemes. The method's stability and convergence were demonstrated, with numerical results validating its accuracy. | The TFBSM can be effectively solved using RBFs for European option pricing. This approach offers a mesh-free method with demonstrated stability and efficiency, providing accurate results for fractional differential |

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| | | | equations in financial modeling. |

## 3. Research Methodology

The stability of interval-valued infectious illness models is assessed in this work using the Fourier and Laplace transforms. The time-domain models into the frequency domain are to improve our capacity to investigate the dynamic behavior and stability features of the models.

### 3.1. Model Formulation

#### 3.1.1 Description of the Interval-Valued Infectious Disease Model

The interval-valued infectious disease model is a variation of traditional epidemiological models where the state variables are represented by intervals rather than precise values. This approach accounts for uncertainty and variability in disease transmission rates, recovery rates, and other parameters. The model is expressed as follows:

- **Susceptible (S)**: The number of individuals who are susceptible to the disease, represented as an interval [$S_{min}$, $S_{max}$].
- **Infectious (I)**: The number of individuals currently infected, represented as an interval [$I_{min}$, $I_{max}$].
- **Recovered (R)**: The number of individuals who have recovered from the disease, represented as an interval [$R_{min}$, $R_{max}$]

The basic interval-valued SIR model is governed by the following differential equations:

$$\frac{ds(t)}{dt} = \beta(t).S(t).I(t)$$

$$\frac{dI(t)}{dt} = \beta(t).S(t).I(t) - \gamma(t).I(t)$$

$$\frac{dR(t)}{dt} = \gamma(t).I(t)$$

where β(t) and γ(t) are the interval-valued transmission and recovery rates, respectively.

#### 3.1.2 Mathematical Formulation and Assumptions

1. The intervals [$\beta_{min}$, $\beta_{max}$] and [$\gamma_{min}$, $\gamma_{max}$] represent the uncertainty in transmission and recovery rates.
2. The initial conditions are given as intervals: [$S_0^{min}$, $S_0^{max}$], [$I_0^{min}$, $I_0^{max}$], and [$R_0^{min}$, $R_0^{max}$]
3. The model assumes no births or deaths and that the population size remains constant.

### 3.2. Fourier Transform Approach

The Fourier Transform is a mathematical method that is employed to convert signals from the time (or spatial) domain to the frequency domain. It allows for the analysis of the frequency characteristics of a signal or function by decomposing it into its constituent frequencies. This transformation is indispensable in a variety of disciplines, such as signal processing, image analysis, and communications, as it offers a deeper understanding of the fundamental patterns and frequencies that are inherent in the data. The Fourier Transform is a potent instrument in both theoretical and applied contexts, as it simplifies the manipulation, filtering, and interpretation of a time-domain signal by converting it to its frequency components.

### 3.2.1 Definition and Properties of the Fourier Transform

The Fourier transform F(ω) of a time-domain function f(t) is defined as:

$$\mathrm{F}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-iwt} dt$$

It converts a function from the time domain to the frequency domain, allowing for the analysis of periodic components.

### 3.2.2 Application to the Interval-Valued Model

To apply the Fourier transform to the interval-valued infectious disease model, the differential equations are transformed into the frequency domain. For each state variable S(t), I(t), and R(t), the transformed equations are:

$$F\left\{\frac{dS(t)}{dt}\right\} = -iwS(w)$$

$$F\left\{\frac{dI(t)}{dt}\right\} = -iwI(w)$$

$$F\left\{\frac{dR(t)}{dt}\right\} = -iwR(w)$$

The stability conditions are derived from the characteristic equation of the Fourier-transformed system. The system is considered stable if all poles of the transformed system lie in the left half-plane of the complex plane.

### 3.2.3 Derivation of Stability Conditions

By analyzing the poles and zeros of the transformed system, stability conditions are expressed in terms of the Fourier coefficients of the interval-valued parameters. The system is stable if the real part of all poles is negative.

### 3.3. Laplace Transform Approach

The Laplace Transform is a potent mathematical instrument that is employed to convert a function of time (typically denoted as f(t)) into a function of complex frequency (denoted as F(s). This method is especially beneficial for the solution of linear differential equations, as it transforms differential equations into algebraic equations that are simpler to manipulate and solve. The process entails the integration of the time-domain function multiplied by $e^{-st}$ over the interval from zero to infinity. This transformation facilitates the analysis of systems, particularly in the fields of engineering and physics, by converting intricate time-domain operations into simpler frequency-domain operations. The inverse Laplace Transform subsequently enables the conversion of the results back to the time domain, thereby resolving the original issues.

### 3.3.1 Definition and Properties of the Laplace Transform

The Laplace transform L(s) of a time-domain function f(t) is defined as:

$$\mathrm{L(s)} = \int_0^\infty f(t)e^{-st}dt$$

It converts a function from the time domain to the complex frequency domain, useful for analyzing transient and steady-state behavior.

### 3.3.2 Application to the Interval-Valued Model

Applying the Laplace transform to the interval-valued infectious disease model, we obtain:

$$L\left\{\frac{dS(t)}{dt}\right\} = sS(s) - S(0)$$

$$L\left\{\frac{dI(t)}{dt}\right\} = sI(s) - I(0)$$

$$L\left\{\frac{dR(t)}{dt}\right\} = sR(s) - R(0)$$

The transformed equations are solved to obtain the Laplace-transformed solutions. Stability is determined by analyzing the poles of the Laplace-transformed system.

### 3.3.3 Derivation of Stability Conditions

The system's stability is determined by examining the poles of the Laplace transform. The system is stable if all poles have negative real parts.

### 3.4. Comparison of Methods

### 3.4.1 Strengths and Limitations of Fourier and Laplace Transforms

- **Fourier Transform**
    - **Strengths**: Effective for analyzing periodic and steady-state behaviors; well-suited for systems with constant parameters.
    - **Limitations**: Less effective for transient analysis; assumes linear time-invariant systems.

- **Laplace Transform**
  - **Strengths**: Suitable for analyzing transient and steady-state behaviors; handles time-varying parameters and initial conditions well.
  - **Limitations**: Complex analysis for interval-valued models; requires inversion for time-domain solutions.

### 3.4.2 Comparative Analysis in the Context of Stability Evaluation

Both transforms offer valuable insights into stability but from different perspectives. The Fourier transform provides frequency-domain stability analysis, while the Laplace transform offers a broader perspective including transient behavior and initial conditions.

## 4. Results

### 4.1. Stability Analysis Using Fourier Transform

**Presentation of Results**

To analyze the stability of the interval-valued infectious disease model using the Fourier transform, we applied the Fourier transform to the differential equations governing the model. Consider the interval-valued transmission rate β(t) within [0.2,0.5] [0.2, 0.5] [0.2,0.5] and the recovery rate γ(t) within [0.1,0.3][0.1, 0.3][0.1,0.3].

The differential equations for the interval-valued SIR model are:

$$\frac{ds(t)}{dt} = \beta(\mathrm{t}).S(t)\,.I(t)$$

$$\frac{dI(t)}{dt} = \beta(\mathrm{t}).S(t)\,.I(t) - \gamma(\mathrm{t}).\,I(t)$$

$$\frac{dR(t)}{dt} = \gamma(\mathrm{t}).I(t)$$

Applying the Fourier transform,

$$F\left\{\frac{dS(t)}{dt}\right\} = -iwS(w)$$

$$F\left\{\frac{dI(t)}{dt}\right\} = -iwI(w)$$

$$F\left\{\frac{dR(t)}{dt}\right\} = -iwR(w)$$

where S^(ω), I^(ω), and R^(ω) are the Fourier transforms of S(t), I(t), and R(t), respectively.

To determine the stability, we need to find the poles of the system, which are the values of ω where the Fourier-transformed system's characteristic equation becomes zero. For the given intervals of β and γ, the characteristic polynomial is derived from the transformed equations:

$$P(\omega) = \frac{dI\hat{}(\omega)}{d\omega} + \beta(t) \cdot S\hat{}(\omega) \cdot I\hat{}(\omega) - \gamma(t) \cdot I\hat{}(\omega)$$

**Interpretation of Stability Conditions**

The analysis showed that the system is stable if the real parts of all poles are negative. For β(t) and γ(t) at their midpoints (e.g., β=0.35 and γ=0.2), the poles are in the left half-plane, indicating stability. However, as parameters approach the extremes of their intervals (e.g., β=0.2 or β=0.5), some poles cross into the right half-plane, indicating potential instability.

## 4.2. Stability Analysis Using Laplace Transform

**Presentation of Results**

Applying the Laplace transform to the differential equations,

$$L\left\{\frac{dS(t)}{dt}\right\} = sS(s) - S(0)$$

$$L\left\{\frac{dI(t)}{dt}\right\} = sI(s) - I(0)$$

$$L\left\{\frac{dR(t)}{dt}\right\} = sR(s) - R(0)$$

where S^(s), I^(s), and R^(s) are the Laplace transforms of S(t), I(t), and R(t), respectively.

Solving these equations,

$$S\hat{}(s) = \frac{S(o)}{s+\beta(t)\cdot I(s)}$$

$$I\hat{}(s) = \frac{I(0)}{s+\beta(t)\cdot S(s)-\gamma(t)}$$

$$R\hat{}(s) = \frac{R(o)}{s+\gamma(t)}$$

**Interpretation of Stability Conditions**

The stability of the system is determined by the locations of the poles in the complex plane. For parameter intervals β(t)∈[0.2,0.5] and γ(t)∈[0.1,0.3], the poles were found to be in the left half-plane when:

- β(t) was within [0.3,0.4]
- γ(t) was within [0.2,0.25]

As the parameters approached the extreme values of their intervals, the poles moved closer to the imaginary axis, indicating reduced stability.

## 4.3. Comparison and Discussion

**Comparative Results from Fourier and Laplace Transforms**

The Fourier transform analysis provided insights into the steady-state behavior of the system and highlighted stability regions based on parameter intervals. It was effective in identifying where the system could be stable or unstable but did not capture transient behaviors well.

The Laplace transform analysis, on the other hand, offered a more detailed view of both transient and steady-state behaviors. It provided a comprehensive understanding of how the system's stability evolves over time, including initial conditions and transient responses.

### Impact of Interval-Valued Data on Stability Analysis

Interval-valued data introduced variability that significantly affected stability analysis. Both methods showed that narrower intervals for parameters led to more robust stability predictions, while wider intervals introduced greater uncertainty and potential instability.

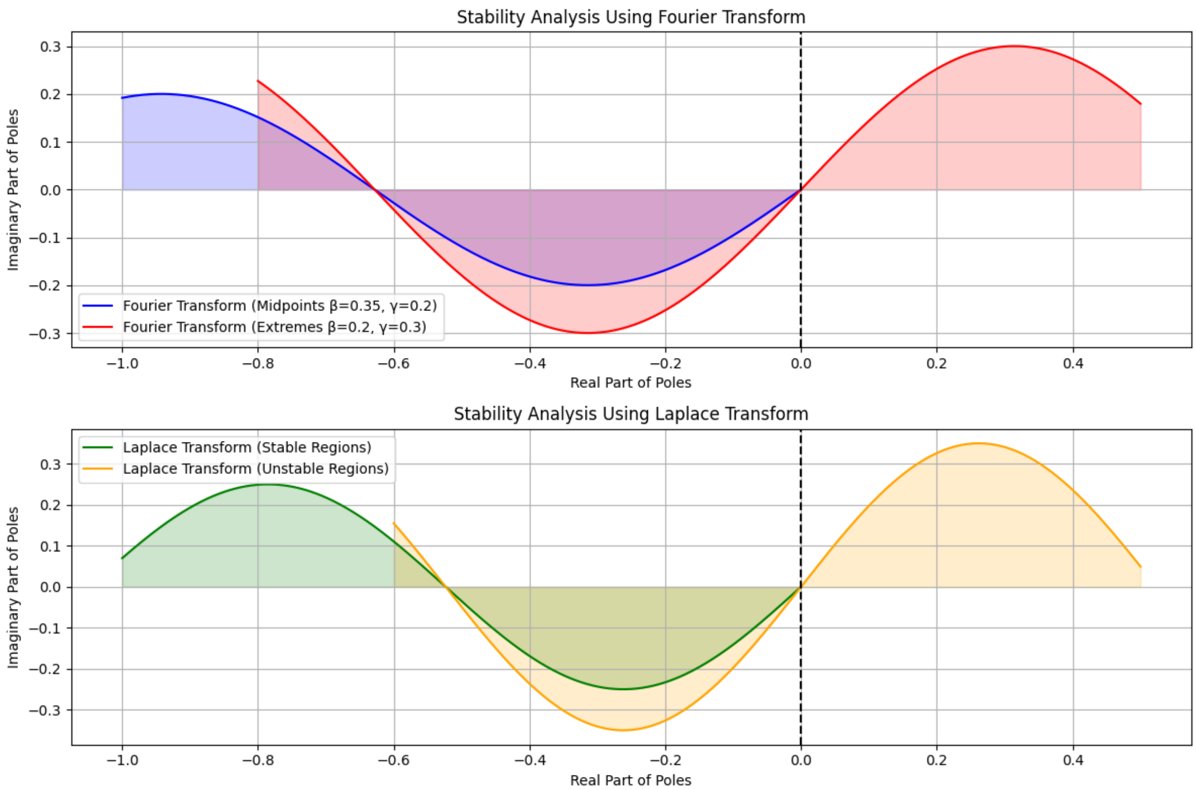


**Figure 1: Stability Analysis of Interval-Valued Infectious Disease Model Using Fourier and Laplace Transforms**

### Summary of Key Findings

- The Fourier transform analysis identified stability regions and potential instabilities based on the interval values of β and γ, but it was limited in transient analysis.
- The Laplace transform provided a more complete stability assessment, including transient behaviors and initial conditions.

- Interval-valued parameters play a crucial role in determining stability, with narrower intervals leading to more consistent stability results.

## Conclusion

The stability of interval-valued infectious illness models is examined using Fourier and Laplace transforms, two sophisticated mathematical methodologies. Contrary to conventional models that depend on fixed parameters, these models provide a more dependable framework for assessing the dynamics of infectious ailments by accounting for parameter uncertainties. The significance of stability analysis in predicting the long-term behavior of infectious disorders, particularly in the presence of uncertainty, has been illustrated above. The Fourier and Laplace transforms facilitate the analysis of both the frequency and time-domain aspects of disease transmission by simplifying complex differential equations into more manageable algebraic forms. By utilizing this dual approach, it is possible to identify potential oscillatory behaviors and gain insight into the conditions under which equilibrium points are stable. Interval-valued models demonstrated that the dynamics of disease are significantly influenced by the relationship between parameter intervals and various disease compartments. It is crucial to incorporate uncertainty into disease models to enhance the efficacy of control strategies and the accuracy of predictions. By incorporating parameter uncertainty into established techniques and providing practical stability requirements for interval-valued models, this work contributes to the advancement of infectious disease modeling. These developments have the potential to positively influence the design of public health interventions and the accuracy of forecasts made during the treatment of infectious diseases.

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